

# Temporal Extensions to Defeasible Logic

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**Abstract.** In this paper, we extend Defeasible Logic (a computationally-oriented non-monotonic logic) in order to deal with temporalised rules. In particular, we extend the logic to cope with durative facts, as well as with delays between the antecedent and the consequent of rules. We showed that the extended temporalised framework is suitable to model different types of causal relations which have been identified by the specialised literature. We also prove that the computational properties of the original logic are still retained by the extended approach.

## 1 Introduction

Non-monotonic logic has been proposed as an appropriate representation of commonsense reasoning. The notion of defeasible conclusion, i.e., a conclusion that can be revised if more evidence is provided, is at heart of commonsense reasoning and therefore of non-monotonic logic. A plethora of non-monotonic formalisms have been investigated, and a common issue is their high computational complexity. To obviate this problem Nute [20] proposed Defeasible Logic. Defeasible Logic (DL) is a rule based non-monotonic formalism that has been designed from beginning to be computationally feasible and easily implementable. Of course there is a trade-off between the expressive power of a logic and its computational complexity. Recently, a number of studies has shown that DL seems to be appropriate to reason in several application areas, ranging from modelling of contracts [13,6], legal reasoning [7,12], modelling of agents and agent societies [11,10], and applications to the Semantic Web [2]. An important finding of these investigations is that time is essential for an accurate representation of real world scenarios. While DL proved to be suitable to cope with most of the phenomena specific to the application domains, the treatment of the temporal issues have been by large ignored, and still is an open problem. Recently a few extensions of DL with time have been proposed [12,9,8].

[12] proposes an extension where each proposition is paired with a timestamp representing the time when the proposition holds. The aim of this work [12] is to study the notion of persistence. On the other hand [9] extends DL by attaching two temporal dimensions to each rules: the first to tell when a rule is deliberated (i.e., when a rule is created), and the second for the time of validity of a rule (i.e. when a rule can be used). In both cases only instantaneous events are considered and there are no explicit temporal relationships between the antecedent and the conclusion of a rule. On the other hand such issues are important not only to model many application domains but also to

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represent theoretical notions such as, for instance, causal relationships. [8] introduces intervals to model deadlines, this permits to have durative events, but there is no explicit temporal relationships between the antecedent and the conclusion of a rule.

In this paper we overcome such limitations by proposing an extension of DL to cope with durative events and with delays between antecedents (causes) and conclusions (effects) of rules in the logic. The proposed formalism also allows for an intensional representation of periodicity. We will first recall the basics of DL (Section 2), then we introduce and motivate the extended temporal formalism (Section 3). Section 4 illustrates the application of the formalism to model some types of causal relations. In Section 5 we formally describe the logic and its inference mechanism. We also show that the proposed extension preserves the computational behaviour of DL.

## 2 Defeasible Logic

Over the years DL [20,3] proved to be a flexible non-monotonic formalism able to capture different and sometimes incompatible facets of non-monotonic reasoning [4], and efficient and powerful implementations have been proposed [18,13].

Knowledge in DL can be represented in two ways: facts and rules. *Facts* are indisputable statements, represented either in form of states of affairs (literal and modal literal) or actions that have been performed. For example, “Tweety is a penguin” is represented by  $Penguin(Tweety)$ . A *rule* describes the relationship between a set of literals (premises) and a literal (conclusion), and we can specify how strong the relationship is. As usual rules allow us to derive new conclusions given a set of premises. For the strength of rules we distinguish between *strict rules*, *defeasible rules* and *defeaters*.

Strict rules, defeasible rules and defeaters are represented, respectively, by expressions of the form  $A_1, \dots, A_n \rightarrow B$ ,  $A_1, \dots, A_n \Rightarrow B$  and  $A_1, \dots, A_n \rightsquigarrow B$ , where  $A_1, \dots, A_n$  is a possibly empty set of prerequisites (causes) and  $B$  is the conclusion (effect) of the rule. We only consider rules that are essentially propositional, i.e., rules containing free variables are interpreted as the set of their ground instances.

*Strict rules* are rules in the classical sense: whenever the premises are indisputable then so is the conclusion. Thus they can be used for definitional clauses. An example of a strict rule is “Penguins are birds”, formally:  $Penguin(X) \rightarrow Bird(X)$ .

*Defeasible rules* are rules that can be defeated by contrary evidence. An example of such a rule is “Birds usually fly”:  $Bird(X) \Rightarrow Fly(X)$ . The idea is that if we know that  $X$  is a bird, then we may conclude that  $X$  flies *unless there is other evidence suggesting that she may not fly*.

*Defeaters* are a special kind of rules. They are used to prevent conclusions not to support them. For example:  $Heavy(X) \rightsquigarrow \neg Fly(X)$ . This rule states that if something is heavy then it might not fly. This rule can prevent the derivation of a “fly” conclusion. On the other hand it cannot be used to support a “not fly” conclusion.

DL is a “skeptical” non-monotonic logic, meaning that it does not support contradictory conclusions. Instead DL seeks to resolve conflicts. In cases where there is some support for concluding  $A$  but also support for concluding  $\neg A$ , DL does not conclude either of them (thus the name “skeptical”). If the support for  $A$  has priority over the support for  $\neg A$  then  $A$  is concluded. No conclusion can be drawn from conflicting rules

in DL unless these rules are prioritised. The *superiority relation* among rules is used to define priorities among rules, that is, where one rule may override the conclusion of another rule. For example, given the defeasible rules

$$r : Bird(X) \Rightarrow Fly(X) \quad r' : Penguin(X) \Rightarrow \neg Fly(X)$$

which contradict one another, no conclusive decision can be made about whether a Tweety can fly or not. But if we introduce a superiority relation  $\succ$  with  $r' \succ r$ , then we can indeed conclude that Tweety cannot fly since it is a penguin.

We now give a short informal presentation of how conclusions are drawn in DL. Let  $D$  be a theory in DL (i.e., a collection of facts, rules and a superiority relation). A *conclusion* of  $D$  is a tagged literal and can have one of the following four forms:

- + $\Delta q$  meaning that  $q$  is definitely provable in  $D$  (i.e., using only facts and strict rules).
- $\Delta q$  meaning that we have proved that  $q$  is not definitely provable in  $D$ .
- + $\partial q$  meaning that  $q$  is defeasibly provable in  $D$ .
- $\partial q$  meaning that we have proved that  $q$  is not defeasibly provable in  $D$ .

Strict derivations are obtained by forward chaining of strict rules, while a defeasible conclusion  $p$  can be derived if there is a rule whose conclusion is  $p$ , whose prerequisites (antecedent) have either already been proved or given in the case at hand (i.e., facts), and any stronger rule whose conclusion is  $\neg p$  has prerequisites that fail to be derived. In other words, a conclusion  $p$  is derivable when:

- $p$  is a fact; or
- there is an applicable strict or defeasible rule for  $p$ , and either
  - all the rules for  $\neg p$  are discarded (i.e., are proved to be not applicable) or
  - every applicable rule for  $\neg p$  is weaker than an applicable strict<sup>3</sup> or defeasible rule for  $p$ .

The formal definitions of derivations and the proof conditions or inference rules for the extension of the logic with the features we are going to describe in the next section will be presented in Section 5.

### 3 Modelling temporal rules

In the following, we first introduce the temporal dimension within the rules and we provide an informal high-level representation of the rules. We then refine it in the next section, showing how high-level rules can be actually modelled within DL. In the discussion below, we use the causal metaphor to explain the relationship between the antecedents and the consequent of the rules. Such a metaphor seems to us quite natural, especially in case rules are interpreted within a temporal framework. We will further investigate the relationships between our temporal rules and different definitions of causation in the literature in Section 4.

We represent rules of the general form:

$$a_1 : d_1, \dots, a_n : d_n \Rightarrow^d b : d_b$$

Such rules have the following general intuitive interpretation:

<sup>3</sup> Notice that a strict rule can be defeated only when its antecedent is defeasibly provable.

- In the rule, all events are represented as a pair  $e : d$ , where  $e$  is the identifier of the event and  $d \in \mathbb{N}, d \geq 1$  is the duration of the event. As special cases, instantaneous events are modelled by  $d = 1$ , while persistent events by  $d = +\infty$ .
- $a_1, \dots, a_n$  are the “causes”. They can start at different points in time.
- $b$  is the “effect”.
- “ $d$ ” ( $d \in \mathbb{Z}$ ) is the *exact* delay between “causes” and “effect”. “ $d$ ” cannot be infinite.

The above definition represents a whole general schema of rules, depending on the choice of the event endpoints which are related by the delay relation. One may consider the following alternative interpretations (where “last” means “temporally last”):

1.  $d$  is the delay between the start of the last cause and the beginning of the effect;
2.  $d$  is the delay between the end of the last cause and the beginning of the effect (in such an interpretation the duration of the causes cannot be infinite).

Other alternatives are technically possible, but seem to us not very motivated from the applicative point of view. For instance, considering  $d$  the delay between the start (or end) of the first cause and the start of the effect seems to us quite counter-intuitive. In fact, if all  $a_1, \dots, a_n$  are explicitly necessary to trigger the rule, it seems quite unnatural that the rule is then triggered as soon as the first of them starts (ends) to hold.

Then to trigger a rule, different conditions have to be satisfied, depending on the chosen interpretation. For instance, let us consider the intuitive triggering conditions for case (2) above (and supposing that all durations are finite).

1. we must be able to prove each  $a_i$  for *exactly*  $d_i$  consecutive time intervals, i.e.,  $\forall i \exists t_0, t_1, \dots, t_{d_i}, t_{d_i+1}$  consecutive time intervals such that we can not prove  $a_i$  at time  $t_0$ , we can prove  $a_i$  at times  $t_1, \dots, t_{d_i}$ , and we cannot prove  $a_i$  at time  $t_{d_i+1}$
2. Let  $t_a^{\max} = \max\{t_{d_j}\} 1 \leq j \leq n$  be the last time when the latest cause can be proven.
3.  $b$  can be proven for  $d_b$  instants starting from time  $t_a^{\max} + d$ , i.e., it can be proven in all the time instants between  $t_a^{\max} + d - 1$  and  $t_a^{\max} + d + d_b + 1$ .

## 4 Causation

Since causation “is common in scientific thinking” and “is dominant in everyday commonsense thinking” [23], it has been widely studied by many AI approaches, mainly aiming to provide computational models of causal reasoning. In particular, many approaches have focused on the relationships between time and causation, starting, e.g., from the pioneering reified logics by McDermott [19] and Allen [1], and by the approaches by Shoham [23], Konolidge [16] and Sandewall [22], that underlined the defeasible character of causal connection and/or stressed the strict relationship between causal reasoning and non-monotonic reasoning (e.g., to deal with change).

Moreover, several approaches focused specifically on the temporal relationships between causes and effects. The problem of determining such temporal constraints is still an open one: for instance, there seems to be no agreement on whether “backward” causation (i.e., causal relations in which the effects can precede the causes) is possible or not (for a collection of accounts on causation by many of the outstanding contemporary philosophers see Sosa [24]).

While domain-dependent approaches in specific application areas have devised restrictive “ad-hoc” definitions (see, e.g., [15] in the medical field), many approaches only identified very general temporal constraint (namely, the constraint that the beginning of the effect cannot precede the beginning of the causes [23,19,1]). An interesting proposal has been devised by Reiger and Grinberg [21], who distinguished between different types of causation depending on the different temporal relations they impose between causes and effects. In particular, in a “One-shot” causal relation the presence of the cause is required only momentarily in order to allow the effect to begin (e.g., to kick a ball causes the ball to move), while in a “Continuous” causal relation the presence of the cause is required in order to begin and sustain the effect (e.g., to run causes the production of lactic acid). Other researchers have followed such a line of research. For instance, within the CYC ontology, Guha and Lenat [14] have identified also “Mutually sustaining” causal relations (e.g., “poverty and illiteracy were mutually sustaining problems plaguing America all during the 1970s”), in which each bit of the cause causes a slightly later bit of the effect, and viceversa (i.e., causes and effects are co-temporal). Analogously, Terenziani and Torasso [25] have introduced also “Culminated Event Causation”, to model causal relationships where the cause is an accomplishment, whose culmination must be reached in order to enforce the effect to start (e.g., switching the light on is an activity with a goal, and its effect –having the light on– is reached with the culmination of such an activity).

The definition of rules we have proposed in Section 3 can model causation, by interpreting the antecedents of the rules as causes and the consequents as effects. Of course, the defeasible non-monotonic character of causation is naturally coped with by the underlying DL. Moreover, the definition of rules we have provided is very general, so that all the above different types of temporal constraints between causes and effects can be captured (as special cases of the general rule) by suitably restricting the values of delay and durations in the rule. For the sake of simplicity, we exemplify such a claim considering simple rules with just one antecedent (cause), i.e., rules of the form  $a : t_a \Rightarrow^d b : t_b$  and considering the interpretation (2) for delays (the case of interpretation (1) is analogous):

- We can model “backward” causation, by considering a negative delay between causes and effects, so that  $0 > t_a + d$ ;
- “one-shot” causation can be modelled with  $0 \leq t_a + d$  and  $0 < t_b + d$ ;
- “continuous” causation can be modelled with  $0 \leq t_a + d$  and  $0 \geq t_b + d$ ;
- “mutually sustaining” causation can be modelled with  $0 = t_a + d$ , and  $0 = t_b + d$ ;
- “culminated event causation” can be modelled simply by imposing  $0 \leq d$ .

## 5 Temporalised Defeasible Logic

As usual with non-monotonic reasoning, we have to specify 1) how to represent a knowledge base and 2) the inference mechanism used to reason with the knowledge base. The language of temporal Defeasible Logic comprises a (numerable) set of atomic propositions  $Prop = \{p, q, \dots\}$ , a discrete totally ordered set of instants of time  $\mathcal{T} = \{t_1, t_2, \dots\}$  and the negation sign  $\neg$ .

We supplement the usual definition of literal (an atomic proposition in *Prop* or the negation of it), with two notions of temporalised literals, one notion to indicate when a literal holds (i.e.,  $p@t$ , where  $t \in \mathcal{T}$ ) and the duration of an event (i.e.,  $p : d$ ,  $d \in \mathbb{N}$ ). Intuitively  $p@t$  means that  $p$  holds at time  $t$ , and  $p : d$  means that  $p$  holds for  $d$  time instants. Given a (temporalised) literal  $p$ ,  $\sim p$  denotes the complement of  $p$ , that is  $\sim p = \neg q$  if  $p = q$ ,  $q \in Prop$ , and  $\sim p = q$  if  $p = \neg q$  for some  $q \in Prop$ .

Formally a *rule*  $r$  consists of its *antecedent* (or *body*)  $A(r)$  ( $A(r)$  may be omitted if it is the empty set) which is a finite set of temporal literals of the form  $p : d$ , an arrow, and its *consequent* (or *head*)  $C(r)$  which is a temporal literal of the form  $p : d$ . The arrow of a rule  $r$  is labelled by a natural number  $d$ , the delay of  $r$ ; we will use  $\delta(r)$  to denote the delay of  $r$ . Given a set  $R$  of rules, we denote the set of strict rules in  $R$  by  $R_s$ , the set of strict and defeasible rules in  $R$  by  $R_{sd}$ , the set of defeasible rules in  $R$  by  $R_d$ , and the set of defeaters in  $R$  by  $R_{df}$ .  $R[q]$  denotes the set of rules in  $R$  with consequent  $q : d$ .

A (temporalised) defeasible theory is a structure  $(\mathcal{T}, t_0, F, R, \succ)$  where:  $\mathcal{T} = \{t_1, t_2, \dots\}$  is a discrete totally ordered set of instants of time;  $t_0 \in \mathcal{T}$  is the reference point (or origin) of  $\mathcal{T}$ ;  $F$  is a finite set of facts, i.e., of temporalised literals of form  $p@t$ ;  $R$  is a finite set of rules;  $\succ$ , the superiority relation, is a binary relation over  $R$  such that the transitive closure of it is acyclic.

A *conclusion* of  $D$  is a tagged literal and can have one of the following four forms:

- $+\Delta q@t$  meaning that  $q$  is definitely provable, at time  $t$ , in  $D$  (i.e., using only facts and strict rules).
- $-\Delta q@t$  meaning that we have proved that  $q$  is not definitely provable, at time  $t$ , in  $D$ .
- $+\partial q@t$  meaning that  $q$  is defeasibly provable, at time  $t$ , in  $D$ .
- $-\partial q@t$  meaning that we have proved that  $q$  is not defeasibly provable, at time  $t$ , in  $D$ .

The proof tags tell us whether it is possible to prove a conclusion at a specific time and the strength or confidence of the conclusion. Provability is based on the concept of a *derivation* (or proof) in  $D$ . A derivation is a finite sequence  $P = (P(1), \dots, P(n))$  of tagged literals satisfying the proof conditions (which correspond to inference rules for each of the kinds of conclusion).  $P(1..n)$  denotes the initial part of the sequence  $P$  of length  $n$ . Before introducing the proof conditions for the proof tags relevant to this paper we provide some auxiliary notions. From now on we will use  $\#$  as a variable ranging over  $\{\Delta, \partial\}$ . All the remaining definition refer to a derivation  $P$ .

**Definition 1.** A temporalised literal  $p : d$  is  $\#$ -active at  $t$  iff 1)  $-\#p@t - 1 \in P(1..n)$ , 2)  $\forall t', t - 1 < t' \leq t + d$ ,  $+\#p@t' \in P(1..n)$ , and 3)  $-\#p@t + d + 1 \in P(1..n)$ . If  $d = +\infty$  then the last condition does not apply.

Intuitively that a literal  $a : d$  is  $\#$ -active at  $t$  means that the event  $a$  started at  $t$  and lasted for  $d$  instants. Based on the notion of  $\#$ -active time we can define the activation time of a rule. The idea is that at the activation time a rule becomes active and trigger its own conclusion.

**Definition 2.** Let  $r$  be a rule. A set  $AT(r) \subset \mathcal{T}$  is an  $\#$ -activation set of  $r$  iff

1. there is a surjective mapping  $\rho$  from  $A(r)$  to  $AT(r)$ , and
2.  $\forall a_i : d_i \in A(r)$ ,  $a_i : d_i$  is  $\#$ -active at  $\rho(a_i : d_i)$ .

$t$  is a #-activation time for  $r$  iff there is a #-activation set  $AT(R)$  s.t.  $t = \max(AT(r))$ .

Since it is possible that a temporal literal  $a : d$  is active at different times, there are many activation times for a rule. The activation time for a rule is used to determine when the rule fires and when the effect of the rule (its consequent) will be produced. The formal relationship between the activation time, the delay and the start of the effect is given in the following definition.

**Definition 3.** A rule  $r$  is #-applicable at  $t$  iff  $t - \delta(r)$  is a #-activation time for  $r$ .

Finally we give a definition of the potential duration of the effect of a rule.

**Definition 4.** A rule  $r \in R_{sd}[p : d_r]$  is  $(t, t')$ -effective iff  $r$  is  $\partial$ -applicable at  $t$ ,  $t' - t < d_r$ .

Let us concentrate on the conditions that prevent a rule to be triggered. Again we start from temporal literals.

**Definition 5.** A temporal literal  $a : d$  is #-passive at time  $t$  iff  $\exists t', t \leq t' < t + d + 1$  such that  $\neg a@t$ .

This means that a temporal literal is passive at time  $t$  if there is an instant in the interval  $t, t + d$  where the literal does not hold.

**Definition 6.** A rule  $r$  is #-discarded at time  $t$  iff  $\exists a_i : d_i \in A(r)$  such that  $\forall t' < t - \delta(r)$ ,  $a_i : d_i$  is #-passive at  $t$ .

We define now a condition to determine whether an applicable rule does not produce its effect since the derivation of the effect is blocked by a stronger rule preventing the effect.

**Definition 7.** A rule  $r \in R_{sd}[p : d_r]$  defeats a rule  $s \in R[\sim p : d_s]$  at time  $t$  iff 1)  $s$  is  $\partial$ -applicable at  $t$ , and 2)  $r$  is  $(t', t)$ -effective, and 3)  $r \succ s$ .

The effect of a rule, i.e., its consequent, has a duration, so the effect can persist. The next definition captures the idea of persistence.

**Definition 8.** A rule  $r \in R_{sd}[p : d_r]$  persists at time  $t$  iff

- $r$  is  $\partial$ -applicable at  $t'$ ,  $t - d_r < t' < t$  and  $\partial p@t' \in P(1..n)$  and
- $\forall t'', t' < t'' \leq t$ ,  $\forall s \in R[\sim p]$  either
  - $s$  is  $\partial$ -discarded at  $t''$  or
  - if  $s$  is  $(t''', t'')$ -effective, then  $\exists v \in R_{sd}[p]$  such that  $v$  defeats  $s$  at  $t'''$ .

We are now ready to give the proof conditions for  $+\Delta$ ,  $-\Delta$ ,  $+\partial$  and  $-\partial$ .

*Proof condition for  $+\Delta$*

If  $+\Delta p@t = P(n+1)$  then

- 1)  $p@t \in F$  or
- 2)  $\exists r \in R_s[p]$ ,  $r$  is  $\Delta$ -applicable at  $t$ , or
- 3)  $\exists r \in R_s[p]$ ,  $r$  is  $\Delta$ -applicable at  $t'$  and  $t - \delta(r) < t'$ .

*Proof condition for  $-\Delta$*

If  $-\Delta p@t = P(n+1)$  then

- 1)  $p@t \notin F$  and
- 2)  $\forall r \in R_s[p]$ ,  $r$  is  $\Delta$ -discarded at  $t$  and
- 3)  $\forall r \in R_s[p]$ , if  $r$  is  $\Delta$ -applicable at  $t'$ , then  $t' + \delta(r) < t$ .

The conditions above are essentially the normal conditions for definite proofs in DL.  $+\Delta$  describes monotonic derivations using forward chaining. The only things to

notice are that: (i) each literal  $a_i : d_i$  in the antecedent must be definitely proved for exactly  $d_i$  times; (ii) clause (3) is a persistence clause and allows us to derive  $p$  at time  $t$  if we have previously proved  $p$  at time  $t'$  and the distance between  $t$  and  $t'$  is less than the duration of a strict rule for  $p$  applicable at  $t'$ .

To prove that a definite conclusion is not possible we have to show that all attempts to give a definite proof of the conclusion fail. In particular we have to check that all strict rules for  $\sim p$  will not produce an effect at  $t$ .

*Proof condition for  $+\partial$*

If  $+\partial p@t = P(n+1)$  then

1)  $+\Delta p@t \in P(1..n)$  or

2).1)  $-\Delta \sim p@t \in P(1..n)$  and

.2)  $\exists r \in R_{sd}[p]$  such that  $r$  persists at  $t$  or  $r$  is  $\partial$ -applicable at  $t$ , and

.3)  $\forall s \in R[\sim p]$  either

.1)  $s$  is  $\partial$ -discarded at  $t$  or

.2) if  $s$  is  $(t', t)$ -effective,

then  $\exists v \in R_{sd}[p]$   $v$  defeats  $s$  at  $t'$ .

*Proof condition for  $-\partial$*

If  $-\partial p@t = P(n+1)$  then

1)  $-\Delta p@t \in P(1..n)$  and

2).1)  $+\Delta \sim p@t \notin P(1..n)$  or

.2)  $\forall r \in R_{sd}[p]$   $r$  does not persist at  $t$  and is  $\partial$ -discarded at  $t$ , or

.3)  $\exists s \in R[\sim p]$  such that

.1)  $s$  is  $(t', t)$ -effective, and

.2)  $\forall v \in R_{sd}[p]$ ,  $v$  does not defeat  $s$  at  $t'$ .

Clauses (1) and (2.1) of  $+\partial$  are the usual clauses of standard DL only relativised to a time instant. In particular clause (1) allows us to weaken a definite conclusion to a defeasible conclusions. Clause (2.1) ensures consistency of the logic, i.e., a stronger proof for the opposite of what we want to prove is not possible. Clause (2.2) requires that at least one rule for  $p$  can indeed produce  $p$  at time  $t$ . Thus we have to subtract the delay from  $t$  to see whether the last cause of the rule started at  $t - d$ . Clause (3.1) is intended to check that each rule for the negation of the conclusion we want to prove is either discarded at  $t$ . i.e., one of the causes of this rule has been proved not to hold for the right duration. Otherwise, an applicable rule for  $\sim p$  producing the effect at  $t$  must be blocked by a stronger rule for  $p$ . The stronger rule should block the beginning of the effect  $\sim p$ .

Let us consider a theory where  $F = \{a@0, b@5, c@5\}$  and with the rules

$$(r_1) a : 1 \Rightarrow^{10} p : 10, \quad (r_2) b : 1 \Rightarrow^7 \neg p : 5, \quad (r_3) c : 1 \Rightarrow^8 p : 5$$

where  $r_3 \succ r_2$ . Based on this theory we can derive  $+\partial p@10$ .  $r_1$  is  $\partial$ -applicable and  $r_2$  is  $\partial$ -discarded. At time 11 we can derive  $p$  since  $r_1$  persists here. At time 12  $r_2$  is no longer discarded, so  $r_2$  interrupts the effect of  $r_1$ . Since we have no means to determine which reason prevails we derive both  $-\partial p@12$  and  $-\partial \neg p@12$ . At 13 rule  $r_3$  becomes active and defeats  $r_2$ , thus from 13 to 18 we can derive  $p$ . However,  $r_3$  terminates its effect at 18, and  $r_1$ , whose duration would ensure that  $p$  persists till 20, cannot be used to derive  $p$  since the effect of  $r_1$  was terminated by  $r_2$  at time 12.

To prove that a defeasible conclusion is not possible we have to show that all attempts to give a defeasible proof of the conclusion fail. In particular we have to check that all rules for  $\sim p$  will not produce an effect at  $t$ , and if they were to produce it there are some active rules for the negation not weaker than rules for the conclusion and the rules are effective at the appropriate instants.

Maher [17] proved that for standard DL the extension of a defeasible theory can be computed in time linear to the size of the theory (where the size of the theory is in

function of the number of symbols and rules in it). Here we can preserve the complexity results for theories without backward causation.

**Theorem 1.** *Let  $D$  be a temporalised defeasible theory without backward causation. Then the extension of  $D$  from  $t_0$  and  $t \in \mathcal{T}$  (i.e., the set of all consequences based on the signature of  $D$  derivable from  $t_0$  and  $t$ ) can be computed in time linear to the size of the theory. i.e.,  $O(|Prop| \times |R| \times t)$ .*

*Proof (sketch).* The idea is to consider snapshots of the theory at every single instant  $t'$  from  $t_0$  and  $t$  to derive the conclusions that hold at time  $t'$ . The rules are linearly reduced to the language of [12]. In particular temporal literals in the antecedent are understood as a set of temporal literals with timestamps (e.g., for  $t'$  a durative literal such as  $a : 2$ , will be expanded to  $\{a@t' - 1, a@t'\}$ ) and literal with infinite duration are dealt with persistent rules. After these steps we can apply the technique of [11] to compute in linear time the extension of the resulting equivalent theory.

Notice that, although the rules contain temporal constraints that are related to STP (Simple Temporal Problem [5]), in this paper, no “classical” form of temporal reasoning (in the sense of temporal constraint propagation [5]) is required. Let us consider the following simple set of rules, involving delays between instantaneous events:

$$\{a : 1 \Rightarrow^{10} b : 1, b : 1 \Rightarrow^{10} c : 1, a : 1 \Rightarrow^{10} c : 1\}$$

In a “classical” temporal constraint environment, such as in the STP framework, the rules in the set would be interpreted as a *conjunction*, so that constraint propagation could be used in order to infer from the first two dependencies that (in case the interpretation (2) is chosen for the delay) the delay between  $a$  and  $c$  is 21, so that the whole set of constraints is not consistent. However, in the present framework, the above set is simply interpreted as a set of *alternative rules*. In such a context, one can consistently say that  $c$  can be caused by  $a$  through the first two rules (where the delay between  $a$  and  $c$  is 21), or directly through the third rule (where the delay is 10).

## 6 Conclusion and Future Extensions

In this paper, we have proposed a temporal extension of DL in order to deal with temporalised durative facts, and with (possible) delays between antecedents and consequences of the rules. The extension we propose

- increases the expressiveness of the logic, as needed to deal with temporal phenomena in several application domains, and, in particular, with different types of causal relationships;
- maintains the computational properties of the original logic. Indeed, the temporal extension has been carefully shaped in order to maintain (in case backward causation rules are not used) the linear complexity of the original logic.

As future work, we aim at analysing the complexity of the extended logic, in case also backward causation rules (i.e., rules in which the consequent can start before the antecedent) are taken into account. The addition of backward rules makes complexity significantly harder, since it requires to navigate the timeline also backward. Our conjecture is that, in such a case, the complexity becomes quadratic on the overall span of time determined by the period of repetition of all the cycles of the rules in the theory.

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