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POPPER ON NECESSITY AND NATURAL LAWS

ABSTRACT: During his philosophical career Popper sought to characterize natural laws alternately as strictly universal and as 'naturally' or 'physically' necessary statements. In this paper we argue that neither characterization does what Popper claimed and sketch a reconstruction of his views that avoids some of their major drawbacks.

1. INTRODUCTION

What distinguishes natural laws from merely 'accidental' generalizations? During his philosophical career Popper provided two answers to this long-debated question. The first claims that the difference between the two kinds of universal statements does not involve any more than the definition of their constitutive terms. The second claims that laws of nature are in some sense 'necessary'. The first answer was put forward in *The Logic of Scientific Discovery* (henceforth *LSD*), in particular §§13–15, and in a short paper published in 1949.[1] The second was proposed in Appendix *x to the 1959 edition of *LSD*.[2] Although these answers may seem incompatible, Popper has maintained that, under certain metaphysical assumptions, they are equivalent. In this paper we shall comment on both Popper's first and second answer (henceforth referred to as the Old and the New Characterization, respectively) and their alleged equivalence. In particular, we shall argue that the Old Characterization fails to provide a basis for any distinction at all between laws of nature and merely accidental generalizations, while the New Characterization succeeds, but at the price of making laws of nature logically necessary and thus indistinguishable from mere tautologies. Furthermore, we shall provide a reconstruction of Popper's views which, in our opinion, avoids these drawbacks.

2. STRICT AND NUMERICAL UNIVERSALITY

Popper's Old Characterization explains the difference between laws of nature and mere universal statements as a distinction between strict and numerical universality. According to Popper, a statement of the form "For all x, if x is a P, then x is a Q" is said to be '*numerically universal*" if P is so defined as to be coextensive with "x is either a, or b, or c, ... ", where $a, b, c, \ldots$ are thought of as names for the elements of the class selected by P. Otherwise it is said to be "*strictly universal*". Thus a strictly universal statement must not be logically

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equivalent to "For all x, if x is either a, or b, or c,..., then x is a Q", and thus to the finite conjunction of singular statements "$Qa \wedge Qb \wedge Qc, \dots$". This would explain the difference between a universal law of nature such as

[S1] All gases expand when heated under constant pressure

and a merely accidental universal such as

[S2] All my friends speak French.

This would also explain why the distinction between laws of nature and merely accidental generalizations cannot be simply accounted for in terms of subjunctive or counterfactual conditionals. For a subjunctive conditional such as

[C1] If x were one of the P's (i.e. either a, or b, or c, ...), then it would be a Q

follows from a strict as well as a numerical universal, while a subjunctive conditional such as

[C2] Is x were added to the P's, then it would be a Q

can only follow from a strict universal. The reason for this is that, in the antecedent of [C1], P retains the same extension it has in the corresponding numerical universal, while in the antecedent of [C2] it is assumed that this extension may change, which obviously makes the inference from a numerically universal statement invalid. (To see this, consider that the extension of P in a numerically universal statement is restricted, and thus in some sense 'closed', while in a strictly universal statement it is unrestricted, and thus in some sense 'open' to the addition of further elements of the same kind of P). This accounts for the fact that, for example, from [S2] it follows intuitively

[C3] If Confucius were a friend of mine, then he would speak French

in the sense of

[C4] If Confucius were (identical with) one of my friends, then he would speak French

but not in the sense of

[C5] If Confucius were added to my friends, then he would speak French.

So far so good. But what about the following statements?

[S3] All planets in our planetary system move in ellipses round the sun

[S4] All the coins in my pocket are silver.

They are both formulated as numerical universals. This notwithstanding, we feel that [S3] is a genuine law of nature, while [S4] is a statement of a merely accidental state of affairs. In effect, according to Popper's suggestion (*NLC*, 64) we may interpret the term "planet of our planetary system" in [S3] in such a way as to make it a law of nature in accordance with our intuition. This can be achieved, for example, by interpreting "planet in our planetary system" as referring (not to the class of the *x*'s which are either Mercury, or Venus, or Earth, or Mars, etc., but) to the class of the *x*'s which are "planets" in all planetary systems similar to our own. Unfortunately, there seem to be no reasons why we could not do the same with the term "coin in my pocket" in [S4], thus making it a law of nature, in contrast to our intuition.

Popper would obviously proscribe such a possibility, for according to his own explicit criterion (*NLC*, 65), for any universal statement *S* and universal term P in it, if *S* is a strictly universal law, then P *can* be interpreted, in the antecedent of the corresponding subjunctive conditional *C*, either as in [C1] or as in [C2]; while if *S* is a, accidental or numerically universal statement, then P *must* be interpreted, in the antecedent of *C*, as in C1. Otherwise, if we interpret P in such a way as to make its extension unrestricted (as in [C2]) we make the inference of *C* invalid, as it should be.

It should be noted that, on Popper's criterion, the interpretation we assign to P in the antecedent of *C* closely depends on our knowing whether *S* is either a strict or a numerical universal. On the other hand, according to this criterion, the distinction between being a law of nature or a mere universal statement is exactly the sort of thing we can know *only* by analyzing the 'structure' of P (*NLC*, 65). Thus Popper's criterion fails to distinguish statements of laws from statements of merely accidental universality, for it presupposes the very distinction it should characterize.

3. FROM UNIVERSALITY TO NECESSITY

The Old Characterization refers only to "the extensional or class-aspect" of the terms (*NLC*, 64)[3] and, according to Popper, makes any explanation of the logical peculiarities of natural laws in terms of a *modal* distinction between statements of necessity and statements of merely accidental universality quite superfluous. Later on, under the influence of Kneale's criticisms,[4] he realized that natural laws are logically stronger than strictly universal statements (*LSD*, 426). Accordingly, Popper turned to the view that his Old Characterization was neither logically sufficient nor intuitively adequate and that it was "quite possible and perhaps even useful" (*LSD*, 428) to speak of the kind of necessity which is manifested in the form of strictly universal natural laws as a sort of 'natural' or 'physical' necessity. Popper proposed the following definition for this kind of necessity (*LSD*, 433):

> (N^0) A statement may be said naturally or physically necessary if, and only if, it is deducible from a statement function which is satisfied in all worlds that differ from our world, if at all, only with respect to initial conditions

where "initial conditions" is the term used by Popper to refer to singular contingent facts. This definition amounts, intuitively, to characterizing natural laws as those statements which hold true in *all* worlds *whatever the initial conditions may be*, which obviously means in all worlds *that share the same natural laws as our own*. This circularity makes no trouble, however, since, as Popper remarks, the above definition

> operates with a perfectly clear intuitive idea — that of varying the initial conditions of our world.... It interprets the result of such changes as the construction of a kind of 'model' of our world...and then imitates the well-known device of calling those statements 'necessary' which are true in...all these models (*LSD*, 435–36).

According to Popper this "device" should define natural or physical necessity as a form of necessity basically distinct from 'ordinary' logical necessity and capable of guaranteeing the necessity of natural laws without obliterating their empirical and contingent character.[5] What Popper seems unaware of is that the "well-known device" he appeals to can hardly be said to define a notion of physical necessity basically distinct from its purely logical counterpart. In fact, his definition (N^0) is nothing but one of the traditional (and unsuccessful) attempts to define physical necessity as a sort of 'relative' or 'conditional' version of logical necessity, that is necessity conditional on the assumption that certain conditions are satisfied (necessity *ex hypothesi*, as Leibniz called it). To put it more formally, Popper's definition lies more or less in the following scheme:

A statement is necessarily true relative to a set of statements T if and only if it is true in all possible worlds in which T holds.

Presumably what, in Popper's view, what would prevent this kind of definition from collapsing into the usual definition of logical necessity (or validity) as truth in all possible worlds, is the fact that it refers not to *all* possible worlds, but to all possible worlds that are assumed to differ from our own only in initial conditions. This may lead one to feel that what has been defined is a separate form of necessity; but this is not the case.[6] We can prove it by recasting Popper's definition (N^0) in the standard language of possible-worlds semantics. In this case we may interpret the usual accessibility (or 'alternativeness') relation in such a way as to make two worlds accessible ('alternative') to each other if and only if they obey the same physical laws.[7]

Obviously, on Popper's own conception, the actual world will be accessible to itself (otherwise how could we ever hope to falsify a law of nature?). Thus by suitably delimiting (by a linguistic *fiat*) the set of relevant possibilities, we can call those statements 'physically' necessary which hold in all the worlds accessible (physically alternative) to our own. In this perspective, Popper's definition (N^0) may be rewritten as

$$(N^0)' \qquad a \vdash_T \Box S \Leftrightarrow \forall w : aRw \models_w T$$

where T is the "appropriate theory",[8] a stands for the actual world and R denotes the relation to its physical alternatives. We can now prove that (N^0) turns out to be equivalent to the usual definition of logical necessity. In fact, since each world is accessible to itself, we have

$$(1) \qquad a \vdash_T \Box S \to S;$$

which, together with the left member of $(N^0)'$ leads to

$$(2) \qquad a \vdash_T S.$$

It is worth noting that, in $a \vdash_T \Box S$ we do not make use of the initial conditions (see section 5. for an explanation). Thus, from (2) we get

$$(3) \qquad \vdash_T S.$$

From this, by the usual soundness result, we obtain

$$(4) \qquad \models_T S$$

from which it follows

$$(5) \qquad \models T \to S,$$

and finally we get

$$(6) \qquad \models_w S.$$

On the other hand, from (3) by the following theorem

THEOREM. For any consistent theory T, $T \vdash A, \Leftrightarrow \forall T_{max} : A \in T_{max}$, where T_{max} denotes a maximal consistent extension of T[9]

we obtain

$$(7) \qquad S \in \mathrm{Cn}(T),$$

where Cn(T) denotes the consequence class of T, for all S's such that, for each maximal consistent extension of T

(8) $$T_{max} \vdash S;$$

but each w is a maximal consistent extension of T, whence

(9) $$\models_w T.$$

We have thus proved that

$(\mathrm{N}^0)''$ $$a \vdash_T \Box S \Leftrightarrow \forall w : aRw \models_w S,$$

which expresses the usual semantic condition for logical necessity, and Popper's N^0, are equivalent.

We have argued that Popper's attempt to define a notion of natural necessity basically distinct from logical necessity is as unsuccessful as most (perhaps all) similar attempts. As far as Popper is concerned, this failure is quite serious for it calls into question the role he assigns to his definition (N^0) in accounting for the necessary character of natural laws. For we agree with Popper that it is *only* workable as a "negative criterion" to show "by finding initial conditions under which the supposed law turns out to be invalid. . . that it was not necessary; that is to say, not a law of nature" (*LSD*, 433). Yet it should be clear that if we do not succeed in distinguishing those statements which are supposed to be necessarily true in the sense of natural or physical necessity (and thus still falsifiable according to Popper's methodological standards) from those which are necessarily true in the sense of logical necessity (i.e. logically or analytically true), then this is bound to remain wishful thinking, causing great trouble for the falsificationist.

4. ON THE EQUIVALENCE BETWEEN THE OLD AND THE NEW CHARACTERIZATION

According to Popper, the Old and the New Characterization differ only in the emphasis the former places on a "metaphysical assumption" which, at the end of *NLC*, he formulates as the

> general principle stating that every kind of event that is compatible with the accepted natural laws does in fact occur in some (finite) space-time region (*NLC*, 66).

Popper appeals again to this principle in Appendix *x where it is referred to as

> the supposition that all logically possible initial conditions (and therefore all events and processes which are compatible with the laws) are somewhere, at some time, realized in the world

to wit that

> our world...comprises all physically possible worlds, in the sense that all physically possible initial conditions are realized in it — somewhere, at some time (*LSD*, 436).

Popper thinks he has eliminated this "metaphysical assumption" from his New Characterization by appealing to "the idea of all worlds that differ (if at all) from our world only with respect to the initial conditions". Yet "once this metaphysical assumption is adopted" the Old and the New Characterization "become (except for purely terminological differences) equivalent, as far as *the status of laws* is concerned" (*LSD*, 436). The "metaphysical assumption" which should make the Old and the New Characterization equivalent is, of course, the venerable metaphysical tenet Arthur O. Lovejoy labelled "the Principle of Plenitude"[10] which goes together with the so-called "statistical interpretation of modality".[11] This interpretation is deeply involved in any attempt to reduce necessity to extensional terms by defining it in terms of universality, for example in Bertrand Russell's contention that necessity is an attribute of propositional functions of which those are necessarily true that are satisfied by all values of their arguments.[12] Now just before his definition (N^0) Popper writes:

> As Tarski has shown, it is possible to explain logical necessity in terms of universality: a statement may be said to be logically necessary if and only if it is deducible (for example, by particularization) from a 'universally valid' statement function; that it to say, from a statement function which is satisfied by every model. (This means, true in all possible worlds). I think that we may explain by the same method what we mean by natural necessity ... (*LSD*, 432).

Thus Popper's definition (N^0) would be meant as a sort of extensional reduction of natural necessity to universality in the wake of Tarski's reduction of logical necessity to universality. This, we may suppose, in Popper's view should argue again for the superfluity of modal notions, for as he emphasizes

> I regard ... necessary as a mere word, as a label, for distinguishing the universality of laws from accidental universality. Of course any other label would do as well, for there is not much connection here with logical necessity (*LSD*, 438).

But what, then, about the phrase "This means, true in all possible worlds" that Popper clearly appends as an explanation of the Tarskian "satisfied by every model"? As Kneale rightly remarked, in Tarski's explanation the (statistically interpreted) modal notions refer to realizations in the actual world and thus, despite Popper's explicit intentions, his proposed explanation is "an explanation of the Leibnizian type",[13]— indeed of the Kripkean type. For, as we have shown above, not only Popper clearly operates with the kind of interpretation of modality codified in what is now generally known as 'possible-world semantics' but, far from there being "not much connection here with logical necessity", what he achieves by its definition (N^0) is not a reduction of necessity to universality but of physical to logical necessity. On the other hand it should be clear that, although there is nothing in the kind of 'possible-world' approach to the logic of necessity which demands that our (or any) world should sometime be actual, once this is assumed the Old and the New Characterization become equivalent as far as the status of laws qua *universal* statement is concerned, which means that the New Characterization would suffer from the same failure to distinguish natural laws from merely accidental generalizations.[14] So if we take Popper's approach seriously the only way of distinguishing natural laws from mere universal statements is by making natural laws necessary in the sense of logical necessity.

5. A (PARTIAL) REHABILITATION OF POPPER'S TWO CHARACTERIZATIONS

As we saw in Section 2 Popper's Old Characterization relies (though circularly) on the idea of using subjunctive conditionals for establishing whether a universal statement S is a natural law or a merely accidental universal. As a first step towards 'rehabilitating' this criterion from the criticisms we put forward in Section 2 we recast it tentatively as follows. Let, as before P be the universal term on which the interpretation of S depends, and C the corresponding subjunctive conditional. Thus

S may be said to be a law of nature if and only if C follows from S under the interpretation of P's extension in the antecedent of C as an unrestricted ('open') class.

and

S may be said to be a statement of accidental universality if and only if C follows from S only under the interpretation of P's extension in the antecedent of C as a restricted ('closed') class.

For the sake of our argument, we need the concept of being T-demonstrable (or being a T-theorem), where $T = \langle B, R\rangle$, B and R being a set of 'axioms' and a set of rules of inference, respectively. For convenience we shall refer to T as a 'theory' and to B as its 'base'. If we wish, we can conceive of B as the set of fundamental principles or 'laws' of T (we shall assume, as usual, that they are statements of the form "For all x, if x is a P, then x is a Q"). On this basis we can define the notion of "being a law of nature relative to T" as follows:

S may be said to be a law of nature relative to T if and only if the consequent of C is T-demonstrable, under the additional assumption of the truth of the antecedent, for the interpretation of P's extension as an unrestricted ('open') *class. Otherwise, S is a mere statement of accidental universality.*

In this sense, for example, [S3] would turn out to be a law of nature relative to a theory T including the fundamental laws of celestial mechanics, for we may suppose that the consequent of the following subjunctive conditional

[C6] If the moon were a planet in our planetary system, then it would move in ellipses round the sun

would turn out to be T-demonstrable under the additional assumption that the moon is a planet (the antecedent of [C4]) via the usual definition of "planet" as

[D1] An opaque celestial body which moves around a star and is illuminated by it

which implies that we interpreted the extension of the term "planet of our planetary system" in the antecedent of [C4] as an unrestricted class. On the other hand [S2] would turn out to be an accidental or numerically universal relative to a theory T', for we may suppose that the consequent of [C3] would not be T'-demonstrable under the additional assumption that Confucius is a friend of mine; although also [C3] turns out to be an analytically true statement under the appropriate theory.

Now let C be the class of all subjunctive conditionals corresponding to a universal statement S. Let us denote by A and B respectively the class of the antecedents and consequents of C, and by $d(A_i)$ the class of elements d in the domain D such that $A_i(d)$. For example, in the case of [S3] we may assume that $d(A_i)$ is the class of celestial bodies we call "planets". We are thus able to say that

(L^0) S may be said to be a law of nature if and only if

(10) $d(A_i) \cup \{x\} \vdash_{T-C_i} B_i(x)$

for an appropriate theory T; otherwise, i.e. if it holds

(11) $d(A_i) \cup \{x\} \nvdash_{T-C_i} B_i(x)$

S is a statement of accidental or numerical universality.

(L^0) makes explicit, in an obvious manner, the idea that a law of nature, as opposed to a mere numerical universal, is not bound by the extension of its domain. It is important to note that the universal statements which are to be characterized as laws of nature according to (L^0) are the same that turn out to be naturally or physically necessary according to (N^0). In other worlds, if we denote by N the class of naturally or physically necessary statements we have, for any universal statement S_i,

$$(12) \qquad d(A_i) \cup \{x\} \vdash_{T-C_i} B_i(x) \Leftrightarrow S_i \in N$$

and, in this sense, the Old and the New Characterization "become . . . equivalent, as far as *the status of laws* is concerned"— even though *this* sense is not that meant by Popper. In fact, the equivalence between (L^0) and (N^0)' implies, via the construction of a Lindenbaum maximal consistent set which produces canonical models, that in a perspective such as Popper's all natural laws are logically necessary, and no law (i.e., no T-theorem) could be falsified — if not, perhaps, accidentally — while Popper himself claimed that a 'genuine theory' must provide theoretical grounds for its falsification. However, this is possible, in a *true* Popperian spirit, only by rejecting the 'mono-theoretical' view underlying his explanation of natural necessity[15] and by assuming a 'constellation' of theories together with some method for comparing them.

A method of this kind could consist in assuming a plurality of theories, even with different rules of inference, as our 'possible worlds' and making them accessible through their bases. This idea may be formalized as follows. Two theories $T_1 = \langle B_1, R_1 \rangle$ and $T_2 = \langle B_2, R_2 \rangle$ are said to be accessible to each other if and only if $P_{B_1} \cap P_{B_2} \neq \emptyset$, where P_{B_i} denotes the set of literals of T_i; this accounts for Popper's idea of "differing, if at all, only with respect to initial conditions". On this basis a statement S may be said to be "naturally" or "physically" necessary if and only if $\vdash_{T_i} S$ for all theories T_i.

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NOTES

[1] K.R. Popper, "A Note on Natural Laws and So-Called 'Contrary-to-Fact Conditionals'", *Mind*, 58 (1949), pp. 62–66. Henceforth referred to as *NLC*).

[2] K.R. Popper, *The Logic of Scientific Discovery*, Hutchinson, London, 1959, and Harper & Row, New York, 1968, pp. 420–41. This appendix was followed by a technical note in 1967: K. R. Popper, "A Revised Definition of Natural Necessity", *Brit. J. Phil. Sci.*, 18 (1967), pp. 316–24, in response to the criticisms put forward in G.C. Nerlich and W.A. Suchting, "Popper on Law and Natural Necessity", *Brit. J. Phil. Sci.*, 18 (1967), pp. 233–35. See the *Addendum, 1968* to Appendix *x in *LSD*, 441.

[3] In view of this 'extensionalistic' approach it is very difficult to see what Popper had in mind

when he characterized strictly universal terms as "intensional" (*NLC*, 65). See also Popper's not very illuminating reference to the intensional aspect of "*genuine universals*" in *Conjectures and Refutations*. The Growth of Scientific Knowlegde, Routledge and Kegan Paul, London, 1969, p. 262.

[4] On the opposing views of Kneale and Popper about necessity *cf.* I. Lakatos, "Necessity, Kneale and Popper", in *Mathematics, Science and Epistemology (Philosophical Papers)*, vol. 2, edited by J. Worral and G. Currie, Cambridge University Press, Cambridge, 1978, pp. 121–27.

[5] In Appendix *x Popper often emphasizes *contra* Kneale that laws of nature are not logically necessary: as "compared with logical tautologies" they have the character of contingent "'merely, accidentally' universal" statements (*LSD*, 429, 432).

[6] This can be said of most (if not all) attempts to define physical necessity by a suitably conditionalized version of logical necessity. From this point of view, it would be interesting to compare Popper's approach with a similar proposal put forward by Richard Montague in 1960: R. Montague, "Logical Necessity, Physical Necessity, Ethics, and Quantifiers", in *Formal Philosophy. Selected Papers of Richard Montague*, edited by R.H. Thomason, Yale University Press, New Haven and London, 1974, pp. 71–83. As Montague explicitly remarks (p. 81), his interpretation of physical necessity amounts to defining it as a kind of relative (i.e. relative to a class K of natural laws) logical necessity.

[7] Some of the work done in conditional logic has gone this direction: see D. Nute, "Conditional Logic", in D. Gabbay and F. Guenthner (eds.), *Handbook of Philosophical Logic, Vol. II*, Reidel, Dordrecht, 1984, p. 393.

[8] In *LSD*, 434 Popper assumes a set of structural true theories about the world, but they can be desumed into our "theory" T.

[9] For the proof of this theorem see E.J. Lemmon (in collaboration with D.S. Scott), *The 'Lemmon Notes': An Introduction to Modal Logic*, edited by K. Segerberg, *American Philosophical Quarterly* Monograph Series, no. 11, Oxford, Basil Blackwell, 1977.

[10] A.O. Lovejoy, *The Great Chain of Being. A Study of the History of an Idea*, Harvard University Press, Cambridge, Mass., 1936.

[11] For the history of this interpretation, see S. Knuuttila (ed.), *Reforging the Great Chain of Being. Studies in the History of Modal Theories*, Reidel, Dordrecht, 1981, and S. Knuutilla (ed.), *Modern Modalities*, Reidel, Dordrecht, 1987.

[12] See B. Russell, "The Philosophy of Logical Atomism", in *Logic and Knowledge. Essays 1901-1950*, edited by R.C. Marsh, Allen & Unwin, London, 1956, pp. 230–31.

[13] W. Kneale, "Universality and Necessity", *Brit. J. Phil. Sci.*, 12 (1961), pp. 100–01.

[14] In fact, as Kneale once remarked, for Popper this assumption "is a direct consequence of his thesis that statements of law are universal material implications; and if it is unplausible, so too is that thesis": see W.C. Kneale, "Natural Laws and Contrary to Fact Conditionals", *Analysis*, 10 (1950), pp. 121–25. Reprinted in *Philosophy and Analysis*, edited by M. Macdonald, Basil Blackwell, Oxford, 1954, p. 250.

[15] See note 8 above and "A Revised Definition of Natural Necessity" (in particular p. 318).